WEAK FACES OF HIGHEST WEIGHT MODULES AND ROOT SYSTEMS

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Abstract. The geometry, and the (exposed) faces, of X a 'Root polytope' or 'Weyl polytope' over a complex simple Lie algebra $\mathfrak{g}$, have been studied for many decades for various applications; by Satake, Borel, Tits, Vinberg, to name a few. This talk focuses on two recent combinatorial analogues to these classical faces, in the discrete setting of weight-sets X.

Chari et al. [*Adv. Math.* 2009 & *J. Pure Appl. Algebra* 2012] introduced and studied two combinatorial subsets of X a root system or the weight-set $\mathrm{wt}V$ of an integrable simple highest weight $\mathfrak{g}$-module V, for studying Kirillov–Reshetikhin modules over the specialization at $q = 1$ of quantum affine algebras $U_q(\hat{\mathfrak{g}})$ and for constructing Koszul algebras. Later, Khare [*J. Algebra* 2016] carefully studied these subsets for $X = \mathrm{wt}V$ for a large class of highest weight $\mathfrak{g}$-modules V, under the names 'weak-$\mathbb{A}$-faces' $\big($for subgroups $\mathbb{A} \subseteq (\mathbb{R}, +)\big)$ and '$(\{2\}; \{1, 2\})$-closed subsets'. For two subsets $Y \subseteq X$ in a vector space, Y is said to be $(\{2\}; \{1, 2\})$-closed in X, if $y_1 + y_2 = x_2 + x_2$ for $y_i \in Y$ and $x_i \in X$ implies $x_1, x_2 \in Y$.

In finite type, Chari et al. classified these discrete faces for X root systems and $\mathrm{wt}V$ for all integrable V, and Khare for all (non-integrable) simple V. In the talk, we will extend and completely solve this problem for *all* highest weight modules V over *any* Kac–Moody Lie algebra $\mathfrak{g}$. We will classify, and show the equality of, the weak faces and $(\{2\}; \{1, 2\})$-closed subsets in the three prominent settings of X: (a) $\mathrm{wt}V \ \forall\ V$, (b) the hull of $\mathrm{wt}V \ \forall\ V$, (c) $\mathrm{wt}\mathfrak{g}$ (consisting of roots and 0). Moreover, they equal in the case of (a) (resp. of (b)) the weights falling on the exposed faces (resp. the exposed faces) of the hulls of $\mathrm{wt}V$.
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