WEIGHT-FORMULAS OF HIGHEST WEIGHT MODULES OVER KAC–MOODY ALGEBRAS

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Abstract. Let $\mathfrak{g}$ be a general complex semisimple or a Kac–Moody Lie algebra, $\mathfrak{h} \subset \mathfrak{g}$ a fixed Cartan subalgebra, and V an arbitrary highest weight $\mathfrak{g}$-module. Recall, the weights of V are the simultaneous eigenvalues (in $\mathfrak{h}^*$) for the $\mathfrak{h}$-action on V. The weights of integrable simple V were classically well-understood, and those of non-integrable simple V were computed recently by Dhillon and Khare [*J. Algebra* 2016, 2022] (in the semisimple & Kac–Moody settings). For general non-simple V, these seem to be unknown even in type A.

In this talk, we will completely solve the problem of determining the weights of arbitrary V. Namely, we will write down a direct, non-recursive, cancellation-free formula for the weights of all highest weight modules V (of all highest weights $\lambda \in \mathfrak{h}^*$) over all general (not necessarily symmetrizable) Kac–Moody algebras $\mathfrak{g}$; this formula is uniform over all $(\mathfrak{g}, \lambda, V)$.

The key ingredients in this weight-formula: working with parabolic Verma modules, and a novel concept called *holes* of V, which we will discuss in the talk. Holes of V are certain independent subsets of the Dynkin diagram nodes of $\mathfrak{g}$, which determine the weights of V.

A curious by-product of our weight-formula is the family of *higher order Verma modules* over $\mathfrak{g}$. These generalize and subsume Verma and parabolic Verma modules, at orders zero and one, respectively. We will introduce these new modules in this talk, and if time permits, we will write down – in certain cases – their character formulas and BGG-type resolutions. Based on joint work with Apoorva Khare (*preprint- arXiv:2203.05515*).