A density function associated to the epsilon multiplicity

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Suppose that I is an ideal in a Noetherian local ring $(R, \mathfrak{m})$ of Krull dimension d. B. Ulrich and J. Validashti defines the ε-multiplicity of I to be

$$\varepsilon(I) := \limsup_{n\to\infty} \frac{\lambda_R\left(H^0_{\mathfrak{m}}\left(R/I^n\right)\right)}{n^d/d!}.$$

This invariant can be seen as a generalization of the classical Hilbert-Samuel multiplicity. S. D. Cutkosky shows that the '$\limsup$' in the definition of ε-multiplicity can be replaced by a limit if the local ring $(R, \mathfrak{m})$ is analytically unramified. An example due to S. D. Cutkosky et al. shows that this limit can be an irrational number even in a regular local ring. Such pathological behaviour occurs because the saturated Rees algebra $\oplus_{n\geq 0}\left(I^n :_R \mathfrak{m}^\infty\right)$ can be non-Noetherian. Throughout this talk, we shall restrict ourselves to homogeneous ideals in a standard graded domain over an algebraically closed field of arbitrary characteristic. Inspired by V. Trivedi's approach to Hilbert-Kunz multiplicity via density functions, we shall introduce a real valued compactly supported continuous function whose integral gives the ε-multiplicity. This function carries a lot more information related to the invariant without seeking extra data. If time permits, we shall produce some explicit examples in low dimensions. This talk will be based on a joint work with S. Roy and V. Trivedi.